A Method of Improving the Response of Waveguide Directional Couplers*

This communication describes a technique for decreasing the frequency variation of coupling of multihole broadwall waveguide directional couplers. The usual curve of coupling vs frequency is shown in Fig. 1[1] and has a peak-to-peak variation of 1.0 db. The proposed method, which is similar to the use of multiple sections in coaxial couplers,[2] perturbs the coupled voltage by adding a small voltage that is in phase at midband where the coupling is looser and out of phase near the band edges. The required perturbation is produced by means of the coupling structure shown in Fig. 2, where the phase of one coupled voltage is delayed with respect to the other coupled voltage because of an added path length ΔL. The coupled power is proportional to:

$$P_c \alpha \left[1 + \left(\frac{k_2}{k_1}\right)^2 + 2\frac{k_2}{k_1}\cos\phi\right] \quad (1)$$

where

$$\phi = \frac{2\pi\Delta L}{\lambda_g}. \quad (2)$$

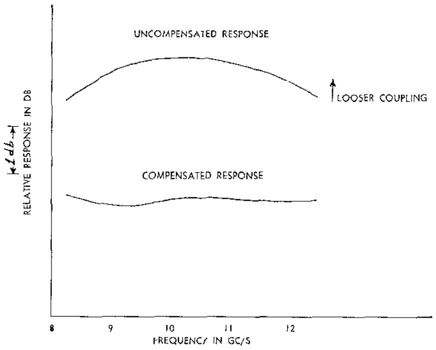

Fig. 1—Compensated and uncompensated coupler response.

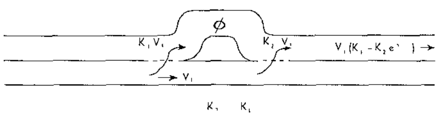

Fig. 2—Proposed method of compensation.

The above equation assumes that the compensating array has the same frequency dependence as does the main array.

The computed compensated response is also shown in Fig. 1 for $\Phi = 360$ at 10.2 Gc and $k_2/k_1 = 0.06$. The original deviation of 0.9 db has been reduced to less than 0.2 db.

One should note that the directivity of the compensating array need not be as great as that of the main array because of its relative decoupling.

LEONARD SWEET
General Microwave Corporation
Farmingdale, N. Y.

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[1] L. I. Kent, "The Optimum Design of Multielement Directional Couplers," Master's thesis, Polytechnic Institute of Brooklyn, N. Y., June, 1954.

[2] E. F. Barnett, P. D. Lacy, and B. M. Oliver, "Principles of Directional Coupling in Reciprocal Systems," presented at the Symposium on Modern Advances in Microwave Techniques, Polytechnic Institute of Brooklyn, N. Y.; November 8–10, 1954.

The Characteristic Impedance of Square Coaxial Line*

Introduction

Recent attempts to design a precision coaxial to strip line transformer have raised the need to know accurately the characteristic impedance of square coaxial line (*i.e.*, a concentric line, as shown in Fig. 1, having inner and outer conductors of square cross section) which forms an intermediate part of the proposed transformer.

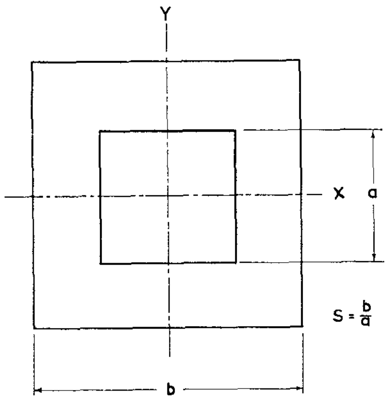

Fig. 1—Cross section of square coaxial line.

Solutions to this problem exist in the literature ranging from the purely empirical [1] through the approximate analytic [2] to the exact [3]. For precise work the empirical and approximate analytic solutions are not generally sufficiently accurate while the exact solution, involving elliptic integrals, is computationally tedious and does not give a direct answer to the usual question of what dimensions must be used to obtain some specified characteristic impedance.

Numerical Solution

Recently as part of a more general task attention has been given to the programming of an IBM 7090 computer to solve boundary value problems in Laplace's equation by the square net method [4]. This effectively reduces the problem of establishing the potentials at the nodal points of the mesh to that of solving a group of simultaneous linear finite difference equations—there being as many equations as meshes in the net. The method of solution is based on the usual matrix techniques but makes use of the special band nature of the matrix to speed inversion. With the present program it is possible to cope with up to about 1000 mesh problems in realistic computing times.

Obviously if the potentials in the space between a pair of conducting surfaces can be established it is a simple matter to extend the work to obtain the charge on the surfaces and hence the capacity. Since capacity per unit length is simply related to the characteristic impedance of the corresponding transmission line by

$$Z_0 = \frac{1}{vC} \quad (1)$$

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where

Z_0 = the characteristic impedance in ohms
C = the capacity per unit length (fd/m)
v = the velocity of light (2.9979×10^8 m/sec)

this method has an obvious application in the solution of the square coaxial line problem.

This has been done for evacuated square coaxial lines of the side length ratios (ratio of outer to inner conductor side length) shown in Table I. An effective increase in the fineness of mesh subdivision for a given program capacity can be had by making use of obvious symmetry and treating only a quadrisection. To further improve the accuracy each problem was worked out three times on an increasingly fine net and the three solutions subjected to Richardson extrapolation [5] to obtain the figure given in the tables.

Accuracy

It is of interest to attempt to estimate the accuracy of this data. It follows from Thomson's theorem [6] that the values of capacity derived in this way must always be too high and hence the characteristic impedances too low. The only way to say by how much would appear to be to compare these solutions with what little numerical data is available from the various analytic attacks.

Chen [2] develops a conformal transformation solution for the capacity of an isolated square corner between two infinite right angle plates and then makes the assumption that a square coaxial line may be considered to be made up of eight paralleled capacities, four parallel plate and four isolated corner. This should give good results for low side length ratios where the corners are well spaced and at the same time make only a small contribution to the total capacity. For a line with $s = 1.25$ Chen's results give $Z_0 = 11.005$ ohms; comparison with Table I indicates agreement to around 0.1 per cent.

Anderson [3] gives an exact conformal transformation solution and uses it to calculate to four figures one numerical example with $s = 3.0$. Interpreted as a characteristic impedance this solution gives $Z_0 = 60.75$ ohms. Comparison with Table I shows a close concurrence within 0.4 per cent. Skiles and Higgins [7] have also attacked this problem by a type of variational approach and also give a numerical result for $s = 3.0$, obtaining somewhat inconsistently with Anderson upper and lower bounds of 60.65 and 60.47 ohms, respectively. The value shown in Table I will be seen to be contained between these limits and to agree with their mean within around 0.05 per cent.

From this evidence it seems fair and perhaps even conservative to claim an average accuracy within $\frac{1}{3}$ per cent for the figures given in Table I.

Application to Design

To make this information more useful for design purposes the basic data given in Table I has been subjected to inverse (Lagrangian) interpolation to prepare a table of side length ratios corresponding to specified characteristic impedances, advancing in one ohm steps from 0 to 100 ohms.